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Integration Technique - Exponentials

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**Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).**

Scan the QR code(s) or click the link for instant detailed model solutions!

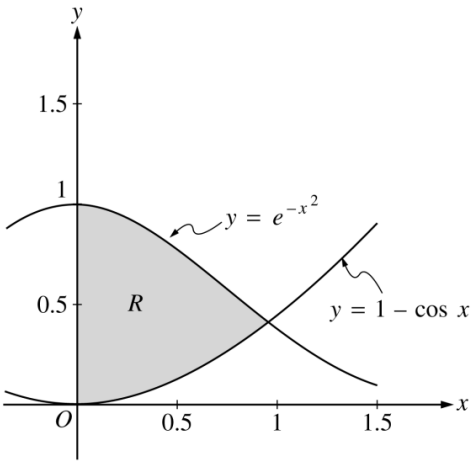
Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Area Between Curves, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Exponentials

Paper: Part A-Calc / Series: 2000 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

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Question 2

Qualification: AP Calculus AB

Areas: Differential Equations

Subtopics: Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Exponentials, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

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Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation, Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Total Amount, Increasing/Decreasing , Integration of Absolute Value Functions, Integration Technique – Exponentials, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2003-Form-B / Difficulty: Hard / Question Number: 4

4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time $t = 3$.
   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
   (c) Find all values of t at which the particle changes direction. Justify your answer.
   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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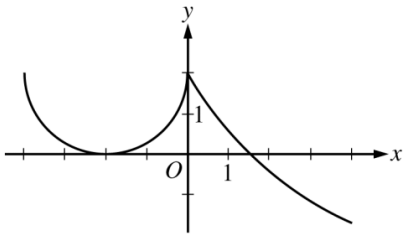
Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Points Of Inflection, Integration Technique – Geometric Areas, Derivative Graphs, Global or Absolute Minima and Maxima, Differentiation Technique – Exponentials, Integration Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: / Question Number: 6



Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}$.

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

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Question 5

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration, Differentiation

Subtopics: Continuities and Discontinuities, Average Value of a Function, Integration Technique – Exponentials, Integration Technique – Trigonometry, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

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6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

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Question 6

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

   (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

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